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WHAT WORKS?
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What kind of mathematics problems help students develop deep, conceptual understanding?

Problem-Based Learning in Mathematics

A Tool for Developing Students' Conceptual Knowledge

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Research Tells Us

- Many students lack a deep understanding of mathematical concepts.
- Classroom teachers find it difficult both to develop a real-life hook for students and to allow students to work through problem solving independently.
- PBL is a promising approach not only to build mathematics understanding but also to test students' conceptual knowledge.
- PBL requires teachers to present students with multifaceted, real-life problems and to act as facilitators supporting students in organizing their own learning.

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Mathematics teachers must teach students not only to solve problems but also to learn about mathematics through problem solving.[1] While "many students may develop procedural fluency … they often lack the deep conceptual understanding necessary to solve new problems or make connections between mathematical ideas."[2] This presents a challenge for teachers: problem-based learning (PBL) provides opportunities for teachers to meet this challenge.

PBL exists as a teaching method grounded in the ideals of constructivism and student-centred learning. When using PBL, teachers help students to focus on solving problems within a real-life context, encouraging them to consider the situation in which the problem exists when trying to find solutions.[3] The majority of research examining PBL focuses on its use in medical schools, with the key features being (a) the use of collaborative small-group work, (b) a student-centred approach, (c) the teacher as facilitator and (d) the use of real-life problems as the organizing focus.[4]

In the medical arena, groups of students are given a set of realistic patient symptoms and expected to research possible diagnoses and courses of treatment; groups work independently, developing and answering their own questions. If, during this diagnostic phase, a group is unsuccessful in addressing key issues, the instructor notes this on their assessment but does not provide the solution.[4] In the classroom setting, it is this aspect of PBL which presents the most signifcant challenge, requiring teachers to shift from direct instruction to supporting students organize their own learning.[5]

From Medical Model to School Classroom

Studies have shown that teachers may have difficulty *not* directing students, *not* determining student progressions and *not* correcting errors.[6,7] For the PBL approach to work, however, teachers need to take on the role of facilitator, encouraging students to work through each problem; this role is "multifaceted and require[s] flexibility" (p. 209).[8]

When starting units using a PBL model, research suggests that elementary teachers find it difficult to develop an appropriate *hook*[8] – a real-life problem that does not have a single or pre-determined solution and, thus, enables students to develop a variety of answers. In this sense, the value of the problem resides in helping students to develop both an understanding of the mathematics and the ability to apply it.[9]

From the authors' case study ...

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A Case Study

The challenges inherent in developing a multifaceted problem and maintaining the teacher's role as facilitator are exemplified in the authors' case study of an Ontario sixth grade teacher who introduced a real-life premise for a follow-up unit on multiplication and percentages. Inviting her students to think of themselves as managers of a new hockey team, she asked them to solve a range of multifaceted problems, only to learn that they had almost no conception of either multiplication or percentages outside of the context of the traditional math unit.

The Problem

Ms. Perry* posed a multifaceted problem that focused on multiplication and percentages. In groups of five, students received the following instructions: "Your next job is developing an 80 game schedule. From the 80 games, 30% have to be from the same division, 15% from the North East Division, 15% from the South East Division and the remaining 40% from the Western Conference." As multiplication and percentages had been covered in a full unit just two weeks prior, the teacher expected students to quickly calculate the number of home games and move on to looking at travel distances. Instead, all groups were stumped.

Trying to Solve the Problem

Students tried a variety of mathematical procedures to come up with a reasonable answer. They tried dividing: 30 went into 80 twice with 20 left over, but 20 did not make sense. They tried calculating the decimal, but 30/80 = .375 and you can not play part of a game. Students were creative in their choices of operations and demonstrated an understanding of what would be considered a reasonable answer: 30% was a little greater than 25% ... given that 25% was the "same" as dividing by four, students knew that a reasonable answer had to be a little greater than 20 ... but how much greater? Many students, when they came up with a number, would try to check it to see if the same procedure, when used with the other percentages, yielded a total of 80 games. Repeatedly, it did not.

As she walked around the room, Ms. Perry was stunned. She was impressed with students' focus on reasonableness, their rechecking of possible solutions, and their perseverance. However, she was shocked at their inability to solve the problem. She repeatedly commented that students had already been tested and had received a C+ or higher. Interviews with students revealed the root of the problem: Context mattered.

The Importance of Knowing "When"

The unit completed by students prior to this task was representative of most math units. Students worked through a textbook and numerous practice sheets on percentages. They practised pulling information out of the written problems and applying the procedures they had learned to come up with their answers. Everything they had done with respect to percentages fell within the percentage unit; there were specific procedures to follow. Students did not have to decide *when* to use a certain procedure.

* All names are pseudonyms.

When Ms. Perry asked the class, "What is 30% of 80?" a few students replied that *of* meant multiplication. However, 30 times 80 did not make sense. Another student suggested using the decimal (0.3). That yielded an answer of 24. This was a reasonable answer. When students calculated the rest of the percentages using the same procedure and revealed a total of 80, they were glad to have the correct answer and moved on to the next part of the task. However, mini-interviews with students revealed a misconception. When asked why they had not tried multiplying by the decimal earlier, the majority of students replied that "Multiplication always makes a bigger number. We needed a number smaller than 80." This response demonstrated that, although students may have understood that 30% represented less than a whole, they did not have a conceptual understanding of multiplication.

Implications for Classroom Practice

Although teachers can implement PBL at the beginning of a unit, using a multifaceted problem to create enthusiasm for learning new knowledge and skills, our study of Ms. Perry's class illustrated that PBL can also be used to check for student misconceptions after a unit of study has been completed.

Classroom Examples

To design your multifaceted problem, focus on identifying where particular mathematical concepts are used regularly by different individuals in society. Try to link the problem with a variety of school curricula.

For Grades K and 1

- Integrate your math PBL with social studies when you study families.
- Use this activity to observe if students know whether to use addition or subtraction.
- After teaching about families, have students draw a picture of everyone in their family.
- With the numbers 0 to 10 written across the bottom of the chalkboard, have students tape their family picture above the number that represents the number of people in their family.
- Have students work in pairs to compare the size of their families. During this activity, be sure not to use the terms "plus" or "minus." See if students know which operation would be useful.
- Have students change partners and come up with questions regarding the family chart (e.g, "How many more people are in Mike's family than Jenna's?"). Students can then share their questions and solutions with the class.

For Grades 2 and 3

- An understanding of adding, multiplying and estimating is required every time we shop at a grocery store. Design a multifaceted problem around shopping in a grocery store.
- Distribute grocery store flyers to each group.
- Have each group calculate how much money they would need to buy enough food (you could link to the Canadian Food Guide) to feed their group for the day.
- Students must calculate amounts of each food item, cost, tax and final total.
- Make a contest of it: Which group can meet the Canadian Food Guide requirements for the day with the lowest budgeted cost?

For Grades 4 and 5

- Students working on measures of central tendency (mean, mode, median) and visual representations of their data can work far beyond just analyzing test scores – have students create their own surveys.
- Link your math class with social studies. Choose a school-wide, municipal or provincial issue.

Using PBL to diagnose student misconceptions ...

- Choose a curriculum objective that you have already taught during the year.
- Imagine real-life situations in which students could use the knowledge and skills associated with those curriculum objectives.
- Have students work in small groups of three or four.
- Ensure that all group members contribute equally by using group-role assignments (e.g., recording the work, handling materials and monitoring group participation) that rotate every 30 to 60 minutes.
- Plan a number of opportunities for all groups to sit together, share progress reports and present questions or concerns; in this way, peers continue to act as sources of information and assistance.

"Research emphasizes the value of problem-based learning for extending student thinking and creativity."

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- In groups of four or five, have students design a survey that uses a numeric-rating scale for answers.
- During lunch and recess breaks, have students administer the survey to other classrooms.
- Have students calculate their measures of central tendency, using a variety of graphs to represent the responses they gathered.
- Conclude by having students share their findings with the rest of the school, either on bulletin boards, during school-wide presentations or in a mini-news report to be given out to each classroom.

For Grades 6 and 7

- Have students use their knowledge of ratios to design a model ice rink.
- Assign students, in groups of four or five, the task of building a Canadian hockey rink to scale. Students can go on the internet to look up the actual dimensions of the rink.
- Provide materials such as styrofoam, paint, popsicle sticks and glue guns. Monitor groups as they construct their ice rinks.
- Watch for common errors such as only scaling one measurement or confusing the idea of scaling with changing units of measure (e.g., switching from metres to centimetres without realizing that they are dividing by 100).
- Integrate this activity with science and let students practise designing electrical circuits by having them add a working light and buzzer.

In Sum

Research emphasizes the value of PBL for extending student thinking and creativity. Multifaceted problems (those that mimic real-life problems and allow a variety of ways to reach a solution) can also be used in the classroom to reveal student misconceptions that traditional tests miss. Our observations of Ms. Perry's class reveal that there is value in having students demonstrate they know when to use specific procedures by working through problems.

References

1. Ontario Ministry of Education (2005). *The Ontario curriculum grades 1 – 8: Mathematics (revised)*. Toronto, ON: Author.
2. Ontario Ministry of Education (2006). *Teaching and learning mathematics: The report of the expert panel on mathematics in grades 4 to 6 in Ontario*. Toronto, ON: Author.
3. Nasir, N.S., Hand, V., & Taylor, E.V. (2008). Culture and mathematics in school: Boundaries between "cultural" and "domain" knowledge in the mathematics classroom and beyond. *Review of Research in Education, 32*, 187–240.
4. Barrows, H.S. (1996). Problem-based learning in medicine and beyond: A brief overview. *New Directions for Teaching and Learning, 68*, 3–12.
5. Grinsven, L.V., & Tillema, H. (2006). Learning opportunities to support student self-regulation: Comparing different instructional formats. *Educational Research, 48*, 77–91.
6. Gallagher, S., Sher, B.T., Stepien, W.J., & Workman, D. (1995). Implementing problem-based learning in science classrooms. *School Science and Mathematics, 95*, 136–146.
7. Hyunju, L., & Sungah, B. (2008). Issues in implementing a structured problem-based learning strategy in a volcano unit: A case study. *International Journal of Science and Mathematics Education, 6*, 655–676.
8. Goodnough, K., & Nolan, B. (2008). Engaging elementary teachers' pedagogical content knowledge: Adopting problem-based learning in the context of science teaching and learning. *Canadian Journal of Science, Mathematics, and Technology Education, 8*, 197–216.
9. Applebee, A.N., Langer, J.A., Nystrand, M., & Gamoran, A. (2003). Discussion-based approaches to developing understanding: Classroom instruction and student performance in middle and high school English. *American Educational Research Journal, 40*, 685–730.

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